HUMAN BODY HEAT ANALYSIS AFTER SOME WORK

Charles Ndung'u

September 2023

1 Introduction

One of the most well-known bioheat transfer equations is Pennes' bioheat equation, which describes the temperature distribution within a tissue due to metabolic heat generation and heat conduction.

$$\rho c \frac{\partial T}{\partial t} = k \nabla^2 T + Q_m - w_b \rho_b c_b (T - T_a) \tag{1}$$

Definations:

- ρ is the density of the material.
- c represents a specific heat capacity.
- $\frac{\partial T}{\partial t}$ is the partial derivative of temperature T with respect to time t, indicating the rate of change of temperature over time.
- k is a thermal conductivity constant.
- ∇^2 represents the Laplacian operator, which is a mathematical operator often used in physics to describe the rate of diffusion or change in a system.
- Q_m represents a heat source or sink term, indicating heat added to or removed from the system.
- w_b is a coefficient related to the interaction between different phases or components in the system.
- ρ_b is the density of a different material or component, often used in multiphase systems.
- c_b is a specific heat capacity associated with the material or component represented by ρ_b.
- T represents temperature.
- T_a is the ambient temperature or a reference temperature.

2 Fourier Series Recarp

$$f(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{2\pi nt}{hour}\right) + b_n \sin\left(\frac{2\pi nt}{hour}\right) \right)$$

Definations of the coefficients

$$a_0 = \frac{1}{hour} \int_0^{1hour} f(t)dt$$

$$a_n = \frac{1}{hour} \int_0^{1hour} f(t) \cos\left(\frac{2\pi nt}{hour}\right) dt$$

$$b_n = \frac{1}{hour} \int_0^{1hour} f(t) \sin\left(\frac{2\pi nt}{hour}\right) dt$$

note that the upper limit of our integral in obtaining the coefficients is one hour as we are interested to see how much heat is generated by our bodies over a period of one hour we choose to use Fourier Series as we are interested in the Heat frequency.
To convert time domain to frequency domain Fourier Series comes in handy!

Odd and Even Functions in Fourier Analysis

Odd Functions:

An odd function is a function $f(x)$ where, for all x in its domain, it satisfies the property: $f(-x) = -f(x)$.

The Fourier series representation for an odd function $f(x)$ is:

$$f(x) = a_0 + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi nx}{T}\right)$$

Even Functions:

An even function is a function $f(x)$ where, for all x in its domain, it satisfies the property: $f(-x) = f(x)$.

The Fourier series representation for an even function $f(x)$ is:

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2\pi nx}{T}\right)$$

In these equations, a_0, a_n, and b_n represent the Fourier coefficients, T is the period of the function, and n is a positive integer representing the harmonic number.

3 Combining the two equations

Combining the Fourier series representation with Pennes' Bioheat Equation involves some complex mathematical steps and thus I will provide an outline, though it's theoretical exercise and actual implementation would require specialized knowledge in both heat transfer and mathematical modeling.

I will assume I have a simplified representation of the temperature variations as a Fourier series:

$$T(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos(2\pi nt) + b_n \sin(2\pi nt)\right)$$

I'll integrate this into Pennes' Bioheat Equation:

$$\rho c \frac{\partial T}{\partial t} = k\nabla^2 T + Q_m - w_b \rho_b c_b (T - T_a)$$

The partial differential equation becomes:

$$\rho c \tfrac{\partial}{\partial t} \left(a_0 + \textstyle\sum_{n=1}^{\infty} \left(a_n \cos\left(\tfrac{T}{2\pi n} t\right) + b_n \sin\left(\tfrac{T}{2\pi n} t\right)\right)\right)$$
$$= k\nabla^2 \left(a_0 + \textstyle\sum_{n=1}^{\infty} \left(a_n \cos\left(\tfrac{T}{2\pi n} t\right) + b_n \sin\left(\tfrac{T}{2\pi n} t\right)\right)\right)$$
$$+ Q_m - w_b \rho_b c_b \left(a_0 + \textstyle\sum_{n=1}^{\infty} \left(a_n \cos\left(\tfrac{T}{2\pi n} t\right) + b_n \sin\left(\tfrac{T}{2\pi n} t\right)\right) - T_a\right)$$

T_a typically represents the arterial blood temperature.

- $Q_m - w_b \rho_b c_b$ is a term in Pennes' Bioheat Equation representing the metabolic heat generation minus the blood perfusion term.
- Q_m represents the metabolic heat generation, which is the heat produced by metabolic processes within tissues.
- w_b is the blood perfusion rate, which represents the volume flow rate of blood per unit volume of tissue.
- ρ_b is the density of blood.
- c_b is the specific heat capacity of blood.
- The term $Q_m - w_b \rho_b c_b$ represents the net effect of metabolic heat generation and the cooling effect due to blood perfusion on the tissue. It quantifies how much heat is being produced by metabolic processes relative to how much is being carried away by blood perfusion.
- The sign of this term depends on the relative magnitudes of metabolic heat generation and blood perfusion. If Q_m is greater than $w_b \rho_b c_b$, the tissue will experience a net increase in temperature, and vice versa.

Solving this equation involves:

- Expanding the derivatives and Laplacian terms.
- Simplifying the resulting expression.

4 Conclusion

Once you obtain the coeficients you will then be able to replace them in the general Fourier Series fomula to obtain the frequency of the heat generated by your body after an hour of intensive labour or work if you will!

So what function of the Fourier series will you use?

answer...in fourier we check symetry in yaxis not x axis

Even Function:

If your heat generation function is even (symmetric about the y-axis), then the Fourier series may consist mainly of cosine terms (even frequencies).

Odd Function:

If your heat generation function is odd (anti-symmetric about the y-axis), then the Fourier series may consist mainly of sine terms (odd frequencies).

Mixed Function:

If your heat generation function has both even and odd components, then both sine and cosine terms will be present in the Fourier series.

Your Task... come up with creative ideas of how we can use your body heat to know your health

Responses... share your responses with me Click here to open in WhatsApp.